Introduction to the THEORY OF ERROR

by

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II. DEFINITIONS

A. Error. This word is used correctly with two different meanings (and frequently incorrectly to denote what properly should be called a "discrepancy"):

(1) To denote the difference between a measured value and the "true" one. Except in a few trivial cases (such as the experimental determination of the ratio of the circumference to the diameter of a circle) the "true" value is unknown and the magnitude of the error is hypothetical. Nevertheless, this is a useful concept for the purpose of discussion.

(2) When a number such as our $\pm 0.000008 \times 10^{10}$ is given or implied, "error" refers to the estimated uncertainty in an experiment and is expressed in terms of such quantities as standard deviation, average deviation, probable error, or precision index. These terms will be defined later.

B. Discrepancy. This is the difference between two measured values of a quantity, such as the difference between those obtained by two students, or the difference between the value found by a student and the one given in a handbook or textbook. The word "error" is often used incorrectly to refer to such differences.

Many beginning students suffer from the false impression that values found in handbooks or textbooks are "exact" or "true." All such values are the results of experiments and contain uncertainties. Furthermore, in experiments such as the determination of properties of individual samples of matter, handbook values may actually be less reliable than the student's because the student's samples may differ in constitution from the materials which were the basis of the handbook values.

C. Random errors. When a given measurement is repeated the resulting values, in general, do not agree exactly. The causes of the disagreement between the individual values must also be causes of their differing from the "true" value. Errors resulting from these causes are called *random* errors. Also, they are sometimes called *experimental* or *accidental* errors. In Section III the types of random error will be enumerated.

D. Systematic or constant errors. If, on the other hand, all of the individual values are in error by the same amount, the errors are called *systematic* or *constant* errors. For example, all the measurements made with a portion of a steel tape which includes a kink will appear to be too large by an amount equal to the loss in length resulting from the kink.

In most experiments, both random and systematic errors are present. Sometimes both may arise from the same source.

E. Determinate and indeterminate errors. Errors which may be evaluated by some logical procedure, either theoretical or experimental, are called *determinate*, while others are called *indeterminate*.

Random errors are determinate because they may be evaluated by application of a theory which will be developed later. In some cases random or systematic errors may be evaluated by subsidiary experiments. In other cases it may be inherently impossible to evaluate systematic errors, and their presence may be inferred only indirectly by comparison with other measurements of the same quantity employing radically different methods. Systematic errors may sometimes be evaluated by calibration of the instruments against standards, and in these cases whether the errors are determinate or indeterminate depends upon the availability of the standards.

F. Corrections. Determinate systematic errors and some determinate random errors may be removed by application of suitable corrections. For example, the measurements which are in error due to a kink in a steel tape may be eliminated by comparing the tape with a standard and subtracting the difference from all the measured values. Some of the random error of this tape may be due to expansion and contraction of the tape with fluctuations of temperature. By noting the temperature at the time of each measurement and ascertaining the coefficient of linear expansion of the tape, the individual values may be compensated for this effect.

G. Precision. If an experiment has small *random* errors, it is said to have high precision.

H. Accuracy. If an experiment has small *systematic* errors, it is said to have high accuracy.

I. Adjustment of data. This is the process of determining the "best" or what is generally called the *most probable value* from the data. If the length of a table is measured a number of times by the same method, by taking the average of the measurements we can obtain a value more precise than any of the individual ones. If some of the individual values are more precise than others, then a weighted average should be computed. These are examples of *adjustment of data* for directly measured quantities. For computed quantities the process may be specialized and complicated. Later we shall develop a method for determining the most probable value of the slope of a straight line representing the graph of linearly related measured quantities.